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Coherent QCD phenomena in the Coherent Pion-Nucleon and Pion-Nucleus Production of Two Jets at High Relative Momenta

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We use QCD to compute the cross section for coherent production of a di-jet (treated as a $q\bar{q}$ moving at high relative transverse momentum, κ_t) from a nucleon and a nuclear target. We find that, in the target rest frame, the space-time evolution of this reaction is dominated by the process in which the high κ_t $q\bar{q}$ component (point like configuration) of the pion wave function is formed before reaching the target. This point like configuration then interacts through two gluon exchange. In the approximation of keeping the leading order in powers of α_s and all orders in $\alpha_s \ln(\kappa_t^2/k_0^2)$, the amplitudes for other processes are shown to be smaller at least by a power of α_s. The resulting dominant amplitude is proportional to $z(1-z)\kappa_t^{-4}$ (z is the fraction light-cone (+) momentum carried by the quark in the final state) times the skewed gluon distribution of the target. For the pion scattering by a nuclear target, this means that at fixed $x_N = 2\kappa_t^2/s$ (but $\kappa_t^2 \to \infty$) the nuclear process in which there is only a single interaction is the most important one to contribute to the reaction. Thus in this limit color transparency phenomena should occur–initial and final state interaction effects are absent for sufficiently large values of κ_t. These findings are in accord with a recent experiment performed at FNAL. We also re-examine a potentially important nuclear multiple scattering correction which is positive and varies as the length of the nucleus divided by an extra factor of $1/\kappa_t^4$. The meaning of the signal obtained from the experimental measurement of pion diffraction into two jets is also critically examined and significant corrections are identified. We show also that for values of κ_t achieved at fixed target energies, di-jet production by the electromagnetic field of the nucleus leads to an insignificant correction which gets more important as κ_t increases.

I. INTRODUCTION

We consider a process in which a high momentum ($\sim$ 500 GeV/c) pion undergoes a coherent interaction with a nucleus in such a way that the final state consists of two jets (JJ) (formed by a $q\bar{q}$ pair) moving at high transverse relative momentum greater than about 2 or 3 GeV. This process was first discussed for both photon and pion projectiles interacting with a nucleon target [1]. In Ref. [2] the possibility of using this process to probe the nuclear filtering of small color dipoles was analyzed. The exclusive JJ production was found to decrease exponentially as κ_t^2 increases due to nuclear filtering. Hence an overall increase of the total diffractive cross sections was suggested as a good signature of the nuclear filtering of small size configurations. In [3] we presented the first application of QCD to this process by generalizing QCD factorization theorems, predicted a nuclear dependence which is qualitatively different from that suggested in [2], and a κ_t dependence which differs by a power of κ_t^2 from [1] and qualitatively from [2]. We also argued that this process can be used to directly measure the $q\bar{q}$ component of the pion distribution amplitude (wave function integrated over transverse momenta). The selection of the final state to be a $q\bar{q}$ pair plus the nuclear ground state causes the $q\bar{q}$ component of the pion wave function to dominate the reaction process. This point is further elucidated in the paper. At very high beam momenta, the pion breaks up into a $q\bar{q}$ pair with large κ_t well before hitting the nucleus and this is justified in the present work. Since the momentum transfer to the nucleus is very small (almost zero for forward scattering), the dominant source of high momentum must be the gluonic interactions between the pion's quark and anti-quark. This is also justified in the present work. Because κ_t is large, the quark and anti-quark must be at small separations–the virtual state of the pion is a point-like-configuration [4]. But the coherent interactions of a color neutral point-like configuration are suppressed (at fixed x_N, $\kappa_t^2 \to \infty$), for processes which involve small transfers of momentum to the target, by the cancellation of gluonic emission from the quark and anti-quark [5,2,4]. Furthermore, the strength of the soft-interaction with the target is proportional to the square of the transverse separation distance between the quark and anti-quark. Thus the interaction with the nucleus is very rare, and the pion is most likely to interact with only one nucleon. The result is that in this process the initial π and the final $q\bar{q}$ pair do not get absorbed by the target, as would typically occur in a low momentum transfer process. Thus initial and final state interactions are suppressed and color transparency unambiguously follows. This treatment of the reaction process in terms of separate wave function and interaction pieces provides a new example of how the QCD factorization theorem works for these kinematics [4]. For this coherent process, the forward scattering amplitude is almost proportional to the number of nucleons, A and the cross section varies as A^2. The forward angular distribution is difficult to observe, so one integrates the angular distribution, and the A^2 variation becomes $\approx \frac{A^2}{R_A^2} \approx A^{4/3}$. This very rapid variation represents a prediction of a very strong enhancement which occurs via the suppression of those strong interaction processes which usually reduce the cross section.

Our interest in this curious process has been renewed recently by exciting experimental progress [6]. Three key predictions of our paper [3] are confirmed by the data:

- The result from the E-791 experiment comparing Pt and C targets is that diffraction for small momentum transfer q_t to the nucleus varies as $\sim A^{1.55\pm0.05}$, which t to the nucleus, is close to our predictions, see Section V. This variation is much stronger than seen in soft diffraction of pions by nuclei $\sim A^{0.8}$ (for a review and references see [4]), and it is qualitatively different from the behavior $\sim A^{1/3}$ of the process suggested in [2].
- The dependence of the cross section $\propto z^2(1-z)^2$ on the fraction of momentum z carried by one of the jets is consistent with our prediction using the asymptotic pion wave function.
- The cross section falls as κ_t^{-n} with $n = 9.2 \pm 0.4$ (stat) $\pm$ 0.3 (sys), also in rough accord with the prediction of $n = -8$ [3].

The purpose of the present work is to re-derive and confirm the earlier theoretical results with a more extensive analysis. The derivation of our leading term [3] directly from QCD by generalizing a QCD factorization theorem of Ref. [7] was presented in Ref. [8], and this is explained more fully now. But here we go further by verifying the assumption that the point-like-configuration is indeed formed well before the projectile reaches the nucleus. This entails explaining that several different amplitudes, which seem to be of the same or lower order in α_s as the one explained above, really are very small.

We also up-date our study of the leading multiple-scattering correction, which is positive [3] and varies as the length of the nucleus divided by an extra factor of $1/\kappa_t^4$ and study the most important competing electromagnetic process. Some specific features of the experimental extraction of the coherent part of the cross section are also explained. Still another feature involves the softer interactions with the target. This was at first derived to be proportional to the gluon density of the nucleus [9]. However, there is a non-zero momentum transfer to the nucleus, so it is actually the skewed gluon density that enters. The skewedness of gluon distribution in the nuclear target leads to a small, calculable correction to the predicted A dependence [10], which changes the detailed nature of our results but not the qualitative features. We find that, in leading log approximation, the absolute value of the differential cross section of diffractive dijet production off nuclei is given by :

$$\frac{d\sigma(\pi + A \to 2jet + A)(q_t = 0)}{dtdzd^2\kappa_t} = \frac{1}{16\pi}\left(\Delta\chi_{pion}(z,\kappa_t)\frac{\pi^2}{3}\alpha_s x_1 G_A(x_1,x_2,\kappa_t^2)\right)^2 \frac{1}{4(2\pi)^3}, \tag{1}$$

where $\chi_\pi(z,\kappa_t) = 4\pi C_F \frac{\alpha_s(\kappa_t^2)}{\kappa_t^2}\sqrt{3}f_\pi z(1-z)$ as is found below, Δ is the Laplacian in κ_t space, z is the fraction light-cone (+) momentum carried by the quark in the final state, and $x_1 G_A(x_1,x_2,\kappa_t^2)$ is the skewed gluon density of the nucleus. x_1, x_2 are the fractions of target momentum carried by exchanged gluons 1 and 2. Note that the resulting κ_t^2 dependence $\propto \kappa_t^{-8}$ is a consequence of a kind of dimensional counting, see discussion in the end of Section II.

It is necessary to discuss the kinematic and dynamic limitations of our analysis. We require high beam energies so that the point-like configuration remains frozen as it passes through the nucleus, and we also require that κ_t be large enough so that the $q\bar{q}$ pair actually be in a point-like configuration. This situation corresponds to κ_t^2/s being held fixed for large values of κ_t^2. For the experiment of Ref. [6] $\kappa_t \approx 2$ GeV, and $s = 1000$ GeV^2, so $x_N = \frac{2\kappa_t^2}{s} \approx .008$. There is another kinematic limit in which κ_t^2 is fixed and s goes to ∞. At sufficiently small values of the ratio $x_N = \frac{2\kappa_t^2}{s} \leq \frac{1}{2m_N R_A}$, the situation is significant different because the quark-anti-quark system is scattered by the collective gluon field of the nucleus. Modifications (enhancement) of the nuclear gluon density actually start at larger values of x_N corresponding to $x_N \sim 1/(2m_N r_{NN}) \sim 0.1$, where $r_{NN} \sim 2$ fm is the mean inter-nucleon distance in nuclei [11,12]. At $x \leq \frac{1}{2m_N R_A}$ this nuclear gluon field is expected to be shadowed, leading to a gradual disappearance of color transparency (at a fixed scale (κ_t^2) characterizing the hardness of the process). This is the onset of perturbative color opacity [3,13]. At even smaller values of x_N a new phenomenon has been predicted – the violation of the QCD factorization theorem [14]. Our present analysis is not concerned with this region of extremely small x_N.

Some general features of our analysis appear in several different Sections, so it is worthwhile to discuss these here. The calculations of several amplitudes are simplified by the use of a general theorem. For each of the graphs of Figs. 1-8, the interaction of the $q\bar{q}$ occurs via two-gluon exchange with the target. The two exchanged gluons are vector particles (bosons) in a color singlet state, so the dominant two-gluon exchange amplitude occurs in a channel which has positive charge and spatial parity, and is therefore even under crossing symmetry. Given this even amplitude, and the condition that we consider high energies $\nu \equiv 2p_\pi m_N$ and fixed small values of the momentum transfer t to the target, we may use the general theorem [15] that

$$\frac{ReA(\nu,t)}{ImA(\nu,t)} = -\frac{\pi}{2}\frac{\partial}{\partial \ln \nu} \ln \frac{ImA(\nu,t)}{\nu}. \tag{2}$$

This means that we can simply calculate the imaginary part of any contribution to the scattering amplitude, with the real part obtainable from Eq.(2). Furthermore, the imaginary part of the scattering amplitude, ImA/ν varies rather slowly with ν, leading to a small value of ImA/ReA, so ImA dominates in the sum of diagrams. The possibility of considering only the imaginary part of the scattering amplitude simplifies the calculations enormously. The relevant intermediate states are on the energy-shell and one can use conservation of four-momentum to relate the momentum of the relevant intermediate states to that of the initial state.

There is another enormous simplification which is related to the issue of gauge invariance. The pion wave function is not gauge invariant, but the imaginary part of the amplitude $\pi + g \to JJ + g$, for two gluons in a color singlet state, is calculable in terms of amplitudes of subprocesses where only one gluon is off mass shell. For such amplitudes the Ward identities [16]–conservation of color current– have the same form as conservation of electromagnetic current in QED. The QED form of current conservation has long been used to simplify calculations of high energy reactions [17], and we shall use that method extensively in what follows.

One more common feature, worthwhile of pointing out here, arises from kinematics. In all of the two-gluon exchange diagrams we consider, Figs. 1-8, the target nucleon, of momentum p, emits a gluon of momentum k_1 and absorbs one of momentum k_2. Conservation of four-momentum gives

$$k_1 - k_2 = p - p' = p_f - p_\pi, \tag{3}$$

in which p' and p_f are the final momenta of the target and the di-jet. Taking the dot product of the above with p_π, for the large pion beam momentum relevant here leads to the relation:

$$x_1 - x_2 = \frac{m_f^2 - m_\pi^2}{2p_\pi \cdot p} = \frac{m_f^2 - m_\pi^2}{\nu}, \tag{4}$$

where

$$x_{1,2} \equiv \frac{k_{1,2}^+}{p^+}. \tag{5}$$

Another immediate consequence of computing only the imaginary part of the amplitude is that

$$x_2 > 0. \tag{6}$$

The last condition is obtained from the requirement that the energies of all almost on-mass-shell quarks in the intermediate states should be positive. Another important consequence of this positivity of energies of particles in the intermediate states6 is that fraction β of the pion's (+) momentum carried by exchanged gluons should satisfy the conditions

$$1 > \beta > 0, \tag{7}$$

for our kinematics. The results (4,6,7) are significant because they will be used in the evaluation of other diagrams. In particular the condition 6 is not fulfilled for the diagrams where transverse momenta of quarks within pion wave function are significantly smaller than observed transverse momenta of jets -see discussion below. Furthermore, the positive nature of x_1, x_2 makes it certain that it is the skewed gluon density which controls the strength of the soft interactions. It is also worth emphasizing that the dominance of small size configurations in the projectile pion, so important to our analysis, is closely related to the renormalizable nature of of QCD. This renormalizability implies, as intensively discussed in the paper, that the selection of large transverse momenta of jets in the final state leads to a selection of the large transverse momenta of the quarks in the pion wave function and to some increase of transverse momenta of the exchanged gluons.

One needs to be aware that in calculating the usual parton density the emissions in the in-state and absorptions in the out-state combine to produce renormalized parton density. So it is necessary to guarantee suppression of gluon radiation collinear to the pion direction in the initial, intermediate and final states. Otherwise the exclusive process will be additionally suppressed by powers of Sudakov type form factor. This is a stringent condition because for small x_N, the time and longitudinal distance intervals ($\sim 1/(2m_N x_N)$) are easily long enough to accommodate the radiation of a gluon.

Consider now impact of these conditions for the interactions of a pion in a large size $q\bar{q}$ configuration. The q and $\bar{q}$ which start off far apart must end up close together in a final state without collinearly moving gluons. In this case, the q and $\bar{q}$ must undergo a high momentum transfer. But such processes are well-known to be exponentially suppressed by Sudakov-type form factors. Only in the case of a compact $q\bar{q}$ pair with a transverse scale commensurate with the gluon virtuality, would the gluon radiation be small. One well-known example of such a suppression is the very small probability to find a pion with q and $\bar{q}$ at *average* distances without a gluon field, if the probe has a resolution $\kappa^2 \gg \Lambda^2_{QCD}$. This is because under these conditions gluons are experimentally observed to carry about $\sim 1/2$ of the pion momentum. Another example is pion scattering by a high momentum gluon field of a target. In the intermediate state there should be a strong collinear radiation along pion direction because of the color charge strongly changes its direction of motion and there is no color charge nearby to compensate for this emission. This is similar to the effect of filling a gap in the case of color unconnected hard processes like Higgs production via $gg \to H$ in hadron-hadron collisions [18].

Here is an outline of the remainder of this paper. Sect. II is concerned with the general formulation and the evaluation of the dominant amplitude (T_1) discussed above, which has the form of the the QCD factorization theorem in the leading order over α_s and all orders in $\alpha_s \ln \frac{\kappa_t^2}{k_0^2}$. The non-leading amplitudes are all evaluated in Sect. III. The first term, T_2, is one in which the hard one-gluon exchange occurs between the q and $\bar{q}$ in the final state. The terms $T_{1,2}$ do not include the effects T_3 which occur when the gluon exchange relevant for high momentum component of the pion wave function interacts with the color fields of the target. Still another set of terms T_4 arise when the hard gluon is exchanged between the $q\bar{q}$ pair of the incident pion during the softer two-gluon exchange with the target. All of the terms $T_{2,3,4}$ are shown to be negligible in the sense that they are smaller than T_1 by at least a power of α_s or by the factor of $\frac{\Lambda^2}{\kappa_t^2}$. In the end of this section we shall demonstrate that the contribution to high κ_t di-jet production resulting from the scattering of a large size q$\bar{q}$ dipole by a large transverse momentum ($\approx \kappa_t$) component of the target gluon field is canceled out in the sum of the relevant Feynman diagrams as a consequence of skewed-ness of parton distributions, and the Ward identities. The nuclear dependence of the amplitude, including a re-assessment of the multiple-scattering correction of [3], is discussed in Sect. IV. Experimental aspects, including the requirements for observing color transparency and the extraction of the coherent cross section are discussed in Sect. V. There is an electromagnetic background term, which becomes increasingly more important as κ_t increases, in which the exchange of a photon with the target is responsible for the diffractive dissociation of the pion. This process, which occurs on the nuclear periphery and is therefore automatically free of initial and final state interactions, is shown in Sect. VI to provide less than a 1 % contribution to the cross section at the values of κ_t and energy of the experiment [6]. A discussion of the implications of observing color transparency as well as a summary and assessment of the present work is provided in the final Sect. VII.

II. AMPLITUDE FOR $\pi N \to NJJ$–EVALUATION OF THE DOMINANT TERM

Let us consider the forward ($t = t_{min} \approx 0$) amplitude, $\mathcal{M}$, for coherent di-jet production on a nucleon $\pi N \to NJJ$ [3]:

$$\mathcal{M}(N) = \langle f(\kappa_t, z), N' \mid \widehat{f} \mid \pi, N\rangle, \tag{8}$$

where $\widehat{f}$ represents the interaction with the target nucleon. The initial $|\pi\rangle$ and final $|f(\kappa_t, z)\rangle$ states represent the physical states which generally involve all manner of multi-quark and gluon components. As discussed in the Introduction, for large enough values of κ_t, only the $q\bar{q}$ components of the initial pion and final state wave functions are relevant in Eq. (8). See also the discussion at the end of section III. This is because we are considering a coherent nuclear process which leads to a final state consisting of a quark and anti-quark moving at high relative transverse momentum. Our notation is that z represents the fraction of the total longitudinal momentum of the beam pion carried by the quark in the final state, and $1-z$ the fraction carried by the anti-quark. The transverse momenta are given by $\vec{\kappa}_t$ and $-\vec{\kappa}_t$. It is necessary to examine the various momentum scales that appear in this problem. The dominant non-perturbative component of the pion wave function carries relative momenta (conjugate to the transverse separation between the q and $\bar{q}$) of the order of $p_t \sim \frac{\pi/2}{\sqrt{2/3}\,(2r_\pi)} \approx 300$ MeV/c. This is much, much lower than the final state transverse relative momenta, greater than about 2 GeV/c. The condition $\kappa_t > 2$ GeV is the minimal requirement to define experimentally the notion of a jet. The immediate implication is that the non-perturbative π wave function, which is approximately a Gaussian, can not supply the necessary high relative momenta. These momenta can only arise from the exchange of a hard gluon, and this can be treated using perturbative QCD.

We continue by letting the $q\bar{q}$ part of the Fock space be represented by $\mid \pi\rangle_{q\bar{q}}$, then the high momentum components can be treated as arising from the following approximate equation [20]:

$$\mid \pi_{q\bar{q}}\rangle = G_0(\pi)V^{\pi}_{eff} \mid \pi_{q\bar{q}}\rangle, \tag{9}$$

where $G_0(\pi)$ is the non-interacting $q\bar{q}$ Green's function evaluated at the pion mass:

$$\langle p_t, y \mid G_0(\pi) \mid p'_t, y'\rangle = \frac{\delta^{(2)}(p_t - p'_t)\delta(y-y')}{m_\pi^2 - \frac{p_t^2+m_q^2}{y(1-y)}}, \tag{10}$$

in which m_q represents the quark mass, y and y' represent the fraction of the longitudinal momentum carried by the quark; and the relative transverse momentum between the quark and anti-quark is p_t. The complete effective interaction V^{π}_{eff}, obtainable in principle from QCD, implicitly includes the effects of all Fock-space configurations. Eq. (10) holds for values of p_t large enough for the denominator to be negative.

A similar equation holds for the final state $q\bar{q}$ pair wave function:

$$\mid f_{q\bar{q}}(\kappa_t, z)\rangle = \mid (\kappa_t, z)\rangle + G_0(f)V^{f}_{eff} \mid f_{q\bar{q}}(\kappa_s, z)\rangle, \tag{11}$$

$$\langle p_t, y \mid G_0(f) \mid p'_t, y'\rangle = \frac{\delta^{(2)}(p_t - p'_t)\delta(y-y')}{m_f^2 - \frac{p_t^2+m_q^2}{y(1-y)} + i\epsilon}, \tag{12}$$

$$m_f^2 \equiv \frac{\kappa_t^2 + m_q^2}{z(1-z)}, \tag{13}$$

in which the first term on the right-hand-side of (11) is the plane-wave part of the wave function. The term m_f^2 is the square of the mass of the di-jet system.

The use of the wave functions (9) and (11) in the equation (8) for the scattering amplitude yields

$$\mathcal{M}(N) = (T_1 + T_2), \tag{14}$$

$$T_1 \equiv \langle(\kappa_t, z), N' \mid \widehat{f} \mid \pi_{q\bar{q}}, N\rangle \tag{15}$$

$$T_2 \equiv \langle f_{q\bar{q}}(\kappa_t, z), N' \mid V^{f}_{eff} G_0(f)\, \widehat{f} \mid \pi_{q\bar{q}}, N\rangle. \tag{16}$$

The term T_2 includes the effect of the final state $q\bar{q}$ interaction; this was not included in our 1993 calculation [3], but its possible importance was stressed in [19]. We shall first evaluate T_1, which is the dominant term.

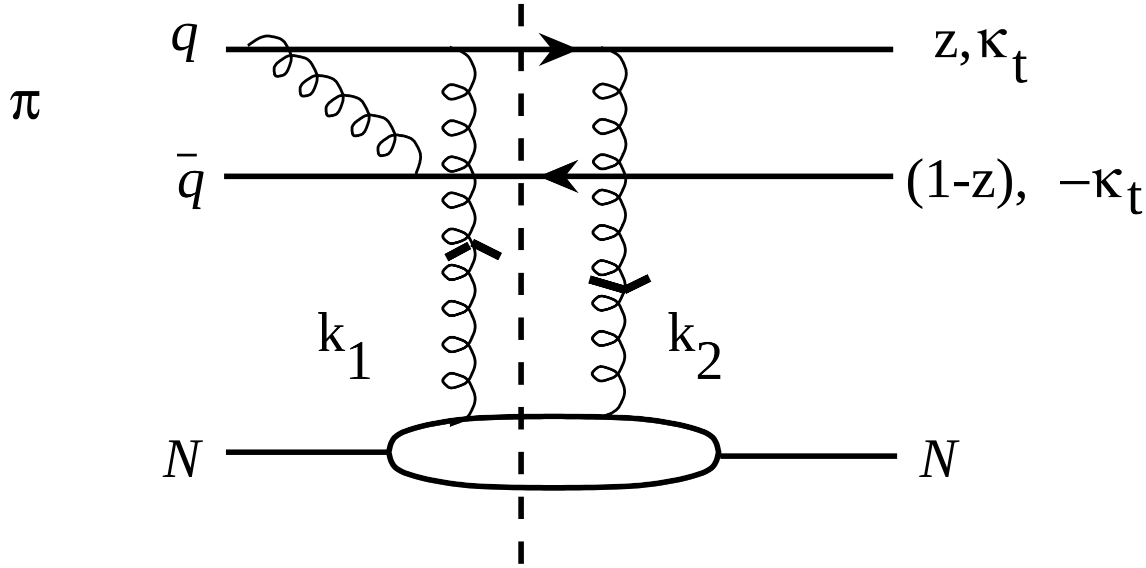

FIG. 1. A contribution to T_1. The high momentum component of the pion interacts with the two-gluon field of the target. The displayed diagram occurs along with its version in which the gluons are crossed. Furthermore, there are four diagrams for each term because each of the gluons can be absorbed or emitted by either the quark or anti-quark of the beam pion. Thus only a single diagram of the eight that contribute is shown.

The evaluation of the graphs corresponding to Fig. 1 consists of two parts. First, we need to know the relevant part of the pion wave function. Second, we need to determine the interaction with the target (here with the gluon field of the target) which causes the pion to disassociate into a $q\bar{q}$.

A. Pion wave function at high momentum transfer

The full wave function $\mid \pi_{q\bar{q}}\rangle$ is dominated by components in which the separation between the constituents is of the order of the diameter of the physical pion. But there is a perturbative tail in momentum space which accounts for the short distance part of the pion wave function. This tail is of dominant importance here because we take the overlap with a final state constructed from constituents moving at high relative momentum. It is therefore reasonable to start our considerations from the one gluon exchange contribution V^g to V^π_{eff} and use light cone gauge $A^+ = 0$, the Brodsky-Lepage [20] normalization and phase-space conventions. Note also that in the calculation of hard high-momentum transfer processes, the $q\bar{q}$ pair in the non-perturbative pion wave function should be considered on energy shell. Corrections to this enter as an additional factor of $\frac{1}{\kappa_t^2}$ in the amplitude.

The perturbative tail is obtained as the result of the one gluon exchange interaction acting on the soft part of the momentum space wave function, defined as

$$\psi(l_t, y) \equiv \langle l_t, y \mid \pi\rangle_{q\bar{q}}. \tag{17}$$

By definition, ψ is dominated by its non-perturbative low-momentum components. However, the amplitude we compute depends on the high momentum tail, χ. For this component, perturbation theory is applicable and we use the one-gluon exchange approximation to the exact $q\bar{q}$ wave function of Eq. (17) to obtain χ, valid for large values of k_t

$$\chi(k_t, x) = -4\pi C_F \frac{1}{\left[m_\pi^2 - \frac{k_t^2+m_q^2}{x(1-x)}\right]} \int_0^1 dy \int \frac{d^2 l_t}{2(2\pi)^3} V^g(k_t, x; l_t, y)\psi(l_t, y) \tag{18}$$

with

$$V^g(k_t, x; l_t, y) = \alpha_s \frac{\bar{u}(x, k_t)}{\sqrt{x}} \gamma_\mu \frac{u(y, l_t)}{\sqrt{y}} \frac{\bar{v}(x, -k_t)}{\sqrt{1-x}} \gamma_\nu \frac{v(1-y, -l_t)}{\sqrt{1-y}} d^{\mu\nu}$$
$$\times \left[\frac{\theta(y-x)}{y-x} \frac{1}{m_\pi^2 - \frac{k_t^2+m_q^2}{x} - \frac{l_t^2+m_q^2}{1-y} - \frac{(k_t-l_t)^2}{y-x}} + (x \to 1-x, y \to 1-y) \right], \tag{19}$$

and $C_F = \frac{n_c^2-1}{2n_c} = \frac{4}{3}$. $d^{\mu\nu}$ is the projection operator of the gluon propagator evaluated in light cone cone gauge. The range of integration over l_t is restricted by the non-perturbative pion wave function ψ. Then in the evaluation of V^g we set m_q and l_t to 0 everywhere in the spinors and energy denominators and evaluate the strong coupling constant at k_t^2. Using

$$\alpha_s(k_t^2) = \frac{4\pi}{\beta \ln \frac{k_t^2}{\Lambda^2}}, \tag{20}$$

where $\beta = 11 - \frac{2}{3}n_f$. Then, if the effects of vertex re-normalization and quark mass re-normalization are included along with the term of Eq. (19), one finds [20]

$$V^g(k_t, x; l_t, y) \approx \frac{\alpha_s(k_t^2)}{x(1-x)y(1-y)} V^{BL}(x, y) \tag{21}$$

where $V^{BL}(x, y)$ is the Brodsky-Lepage kernel:

$$V^{BL}(x, y) = 2\left[\left(\theta(y-x)x(1-y) + \frac{\Delta}{x-y}\right) + (x \to 1-x, y \to 1-y)\right], \tag{22}$$

with the operator $\frac{\Delta}{x-y}$ defined by its action on a function of y and k_t^2 as $\frac{\Delta}{x-y}\phi(y, k_t^2) = (\phi(x, k_t^2) - \phi(y, k_t^2))/(x-y)$. The net result for the high k_t component of the pion wave function is then

$$\chi(k_t, x) = \frac{4\pi C_F \alpha_s(k_t^2)}{k_t^2} \int_0^1 dy V^{BL}(x, y) \frac{\phi(y, k_t^2)}{y(1-y)}, \tag{23}$$

where

$$\phi(y, k_t^2) \equiv \int \frac{d^2 l_t}{(2\pi)^3} \theta(k_t^2 - l_t^2) \psi(l_t, y). \tag{24}$$

The analysis of experimental data for virtual Compton scattering and the pion form factor performed in [27,28] shows that this amplitude is not far from the asymptotic one for $k_t^2 \geq 2-3$ GeV^2

$$\phi(k_t^2 \to \infty, x) = a_0 x(1-x), \tag{25}$$

where $a_0 = \sqrt{3} f_\pi$ with $f_\pi \approx 93$ MeV.

Equation (23) represents the high relative momentum part of the pion wave function. Using the asymptotic function (25) in Eq. (23) leads to an expression for $\chi(k_t, x) \propto x(1-x)/k_t^2$ which is of the form used in Ref. [3]. In particular,

$$\chi(k_t, x) = \frac{4\pi C_F \alpha_s(k_t^2)}{k_t^2} a_0 x(1-x). \tag{26}$$

This result for $\chi(k_t, x)$ is valid with a precision $1 + \mathcal{O}(\alpha_S)$.

B. Interaction with the target

To compute the amplitude T_1, it is necessary to specify the scattering operator $\widehat{f}$. For high energy exclusive scattering processes the operator $\widehat{f}$ changes only the transverse momentum, and therefore in the coordinate space representation $\widehat{f}$ depends on b^2. The transverse distance operator $\vec{b} = (\vec{b}_q - \vec{b}_{\bar{q}})$ is canonically conjugate to $\vec{\kappa}_t$. At sufficiently small values of b, the leading twist effect and the dominant term at large s arises from the diagrams when pion fragments into two jets as a result of interactions with the two-gluon component of gluon field of a target, see Figure 1. The perturbative QCD determination of this interaction, which is a kind of the QCD factorization theorem, involves a diagram similar to the gluon fusion contribution to the nucleon sea-quark content observed in deep inelastic scattering. One calculates the box diagram for large values of κ_t using the wave function of the pion instead of the vertex for $\gamma^* \to q\bar{q}$. The application of the technology leading to the QCD factorization theorem in the impact parameter space leads [4,21,22] to

$$\widehat{f}(b^2) = is\frac{\pi^2}{3}b^2\left[x_N G_N(x_N, Q^2_{\rm eff})\right]\alpha_s(Q^2_{\rm eff}), \tag{27}$$

in which G_N is the gluon distribution function of the nucleon, and $Q^2_{\rm eff} = \frac{\lambda^2}{b^2}$. For our kinematics, it is reasonable to use $\lambda(x = 10^{-3}) = 9$ [14]. The formula (27) is actually not complete. The mass difference between the pion and the final two-jet state requires that the reaction proceed by a non-zero momentum transfer to the target. This means that the function G_N should be replaced by the skewed (or off-diagonal or generalized) gluon distribution. Thus the distribution function should depend on the plus components x_1, x_2 of the momenta k_1, k_2 of the two exchanged gluons, Eq. (5).

The difference between the skewed and ordinary gluon distribution is calculable in QCD using the QCD evolution equation for the skewed parton distributions [23,24]. The kinematical relation between x_1 and $x_1 - x_2$ is given in Eq. (4). But x_1 is close to x_N of Eq. (28), while x_2 is small [23,24] in the calculation of T_1. The skewed parton distribution can be approximated by gluon distribution [25,26] if

$$x_N \approx (x_1 + x_2)/2 \approx \frac{\kappa_t^2}{2z(1-z)s}. \tag{28}$$

While including the effect of skewed-ness would alter any detailed numerical results, the qualitative features of the present analysis would not be changed.

The most important effect shown in Eq. (27) is the b^2 dependence, which shows the diminishing strength of the interaction for small values of b. For the sake of brevity it is convenient to rewrite σ in the form:

$$\widehat{f}(b^2) = is\frac{\sigma_0}{\langle b_0^2\rangle}b^2 = is\frac{\sigma_0}{\langle b_0^2\rangle}\left(-\nabla^2_\kappa\right) \tag{29}$$

in which the logarithmic dependence of α_s on b^2 may be included in σ_0. Our notation is that $\langle b_0^2\rangle$ represents the pionic average of the square of the transverse separation, and

$$\frac{\sigma_0}{\langle b_0^2\rangle} \approx \frac{\pi^2}{3}\alpha_s(\kappa_t^2)[x_N\, G_N^{(skewed)}(x_1, x_2, \kappa_t^2)], \tag{30}$$

in which the ordinary gluon distribution is used.

The above result (30) holds for x_N about 10^{-2}. For x_N of about 10^{-3} or smaller, the second kinematic regime mentioned in the introduction is relevant, and one would obtain different results. For still smaller values of x, say $x \sim 10^{-5}$ non-linear gluonic effects become important and the present treatment of the $q\bar{q}$ interaction with the target will be insufficient.

C. One Gluon Exchange in the Pion– T_1

The necessary inputs to evaluating T_1 of Eq. (15) are now available. The approximate pion wave function, valid for large relative momenta, is given by Eq. (26). The interaction $\widehat{f}$ is given by Eq. (29). The use of Eq. (29) allows a simple evaluation of the scattering amplitude T_1 because the b^2 operator acts on the pion wave function (here σ_0 is treated as a constant) as $-\nabla^2_{\kappa_t}$. Using Eqs. (9) and (29) in Eq. (15), leads to the result:

$$T_1 = -4is\frac{\sigma_0}{\langle b^2\rangle}\frac{4\pi C_F\alpha_s(\kappa_t^2)}{\kappa_t^4}\left(1 + \frac{1}{\ln\frac{\kappa_t^2}{\Lambda^2}}\right) a_0\, z(1-z) \tag{31}$$

This amplitude T_1 is (except for the factor in the parentheses) of the same form as our 1993 result [3]. The present result is obtained directly from QCD, in contrast with the earlier work which used some phenomenology for the pion wave function.

Note that the factor in parenthesis is close to unity (it is 1.22 if $\kappa_t = 2$ GeV and $\Lambda \approx 0.2$ GeV). This small correction arises unambiguously from the application of Eq. (29), but there are a number of other logarithmic corrections which we have not considered. Thus the size of the logarithmic correction term should be understood only as a rough indication of the error involved in using Eq. (31). We stress that the essential dependence of T_1 is as $\frac{1}{\kappa_t^4}$, and note in passing that the amplitude suggested in Ref. [2] varies as a Gaussian in κ_t. It is easy to check that Eq. (31) is valid if the leading order (LO) QCD evolution over κ_t^2 of the pion wave function is included. This is because the LO QCD evolution corresponds to a strong ordering of the transverse momenta of partons.

Our κ_t dependence of T_1 leads to: $\frac{d\sigma(\kappa_t)}{d\kappa_t^2} \propto \frac{1}{\kappa_t^8}$ for $x_N \sim 10^{-2}$. This can be understood using simple reasoning. The probability to find a pion at $b \leq \frac{1}{\kappa_t}$ is $\propto b^2$, while the square of the total cross section for small dipole-nucleon interactions is $\propto b^4$. Hence the cross section of productions of jets with sufficiently large values of κ_t is $\propto \frac{1}{\kappa_t^6}$, for $x_N \sim 10^{-2}$, leading to a differential cross section $\propto \frac{1}{{\kappa_t}^8}$. This reasoning ignores the dependence of the gluon structure function on κ_t. For sufficiently small values of $x (x \sim 10^{-3})$, gluon evolution would give a a somewhat different behavior.

III. OTHER AMPLITUDES

So far we have emphasized that the amplitude we computed in 1993 is calculable using perturbative QCD. However there are four other types of contributions which occur at the same order of α_s. The previous term in which the interaction with the target gluons follows the gluon-exchange represented by the potential V^g in the pion wave function has been denoted by T_1. But there is also a term, in which the interaction with the target gluons occurs before the action of V^g is denoted as T_2, see Figure 2. However, the two gluons from the nuclear target can also be annihilated by the exchanged gluon (color current of the pion wave function). This group of amplitudes , denoted as T_3, is shown in Figure 3. The sum of diagrams when one target gluon is attached before the potential V^g and a second after the potential V^g, see E.g. Figure 4, corresponds to an amplitude, T_4.

We discuss each of the remaining terms T_2, T_3, T_4 in the following sub-sections and will show that they give negligible contributions for meson but not photon projectiles. A previous work [8] reported that a preliminary investigation found that T_2 could be significant. That result is not supported here; we shall find T_2 to be negligible.

A. One Gluon Exchange in the Final $q\bar{q}$ Pair–T_2

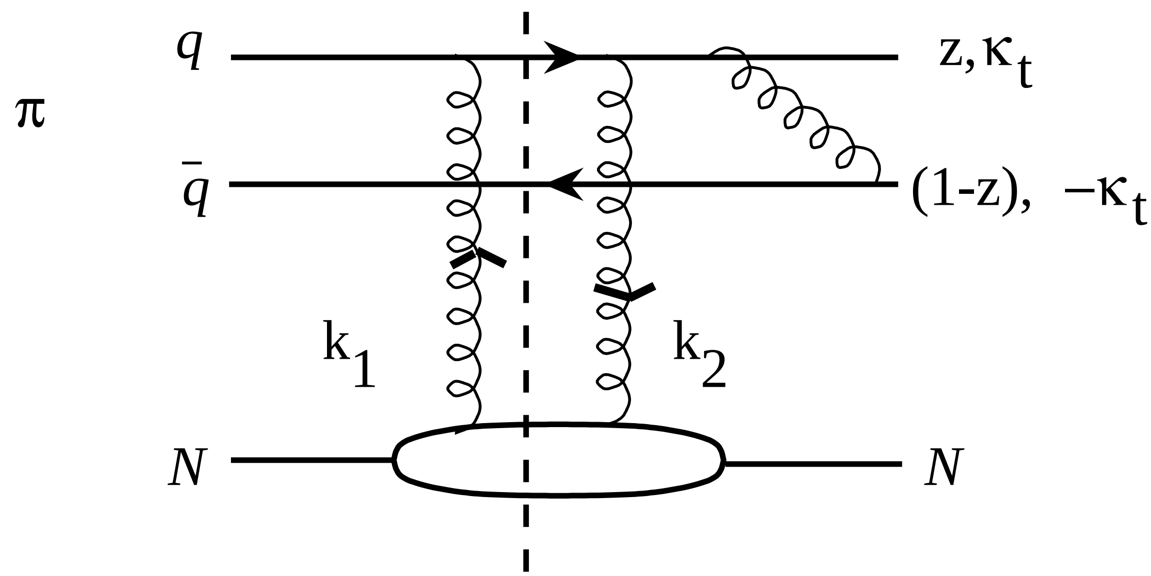

FIG. 2. Contribution to T_2. The high momentum component of the final $q\bar{q}$ pair interacts with the two-gluon field of the target. Only a single diagram of the eight that contribute is shown.

The existence of the term T_2, defined in Eq. (16) and displayed in Fig. 2, caused Jennings & Miller [19] to worry that the value of $\mathcal{M}_N$ might be severely reduced due to a nearly complete cancellation. At first glance, the graph seems difficult to evaluate because of the appearance of the pion's soft non-perturbative wave function. However, we can show that this term vanishes in leading order in α_s and all orders in $\alpha_s \ln \kappa_t^2/k_0^2$. Since we are interested in the leading power of κ_t^2 we will neglect m_π^2, m_q^2 as compared to κ_t^2.

We need to evaluate only the imaginary part of the diagram (and use Eq. (2) to get any necessary real part). There are two kinematic regimes to consider. The first has $l_t \ll \kappa_t$, $k_{1t} \ll \kappa_t$, and the second $l_t^2 \sim k_{1t}^2 \sim \kappa_t^2$. Here l_t is the quark transverse momentum within the pion wave function. We consider the former regime first, as it is expected to be more important. In this case, we shall employ conservation of four-momentum to argue that T_2 must vanish. Conservation of four-momentum can be used to relate the intermediate state (denoted by the vertical dashed line, occurring between the emission and the absorption of the gluons by the target in the diagram of Fig. 2 [30]) of momentum $\tilde{p}$ with $\tilde{p}^2 \equiv \tilde{m}^2$ with the intermediate state. The mass of the intermediate state is given by

$$\tilde{m}^2 \approx x_1\nu - x_1\beta\nu - k_{1t}^2, \tag{32}$$

where β is the light-cone fraction of the pion momentum carried by an exchanged gluon: $\beta = \frac{-k_1^-}{p_\pi^-} = \frac{-k_2^-}{p_\pi^-}$. Thus we arrive at the equation:

$$x_1 = \frac{\tilde{m}^2 + k_{1t}^2}{(1-\beta)\nu}. \tag{33}$$

It follows from the requirement of positivity of energies of all produced particles in the intermediate states that $0 \leq \beta \leq 1$. We can now calculate $\tilde{m}^2$ directly through the light cone momenta of $q\bar{q}$ pair in the intermediate state:

$$\tilde{m}^2 = \left(\frac{l_t^2}{z} + \frac{(k_{1t}-l_t)^2}{1-z-\beta}\right)(1-\beta) - k_{1t}^2. \tag{34}$$

Combining Eqs. (32),(34) we obtain

$$\frac{l_t^2}{z} + \frac{(k_{1t}-l_t)^2}{1-\beta-z} = x_1\nu, \tag{35}$$

which, when using Eq. (4) leads to

$$x_1 = \frac{1}{\nu}\left(\frac{l_t^2}{z} + \frac{(k_{1t}-l_t)^2}{1-\beta-z}\right) = \frac{m_{2jet}^2}{\nu} + x_2, \tag{36}$$

Therefore

$$x_2 = \frac{1}{\nu}\left(\frac{l_t^2}{z} + \frac{(k_{1t}-l_t)^2}{1-\beta-z} - \frac{\kappa_t^2}{z(1-z)}\right). \tag{37}$$

In order for the term T_2 to compete with T_1 we need to have $l_t \ll \kappa_t$, $k_{1t} \ll \kappa_t$. These kinematics cause Eq. (37) to yield the result: $x_2 < 0$ and therefore no contribution to the imaginary part of the amplitude is possible.

This argument can be carried out for all combinations of diagrams represented by Fig. (2). For example, another attachment of gluons, in which the gluon k_1 is absorbed by the quark, corresponds to interchanging z with $1-z$, and therefore leads to the same vanishing result.

Now we consider the second situation: $l_t^2 \sim k_{1t}^2 \sim \kappa_t^2$. This is the typical situation in which there are extra hard lines, as compared with the dominant terms, and one obtains a suppression factor $\sim 1/\kappa_t^2$. We can demonstrate this suppression using another technique which is based on the Ward identity [16]. This technique is used extensively in our subsequent discussion of the T_3 contribution. The vanishing of T_2, in leading logarithm approximation, for our second situation follows immediately using that technique.

The net result of all of this is that the final state gluon exchange diagrams of Fig. 2, can be ignored. This additional suppression of T_2 as compared with the earlier estimates [19] reflects the detailed nature of the two-gluon exchange interaction and the mismatch between the low mass of the intermediate state $\tilde{m}$ and the much higher values $\sim m_f$ characteristic of the intermediate state of Fig. 1. One may also say that the characteristic distance scale related to Fig. 2 $(2p_\pi/\tilde{m}^2)$ is much larger than that $(2p_\pi/m_f^2)$ related to Fig. 1. The exchange of a single relatively soft gluon can not efficiently connect states associated with such very different distance scales.

Note that the Feynman diagram corresponding to Fig. 2 also contains the time ordering of Fig. 3, in which the $q\bar{q}g$ configuration interacts with the target. In taking the imaginary part of the amplitude, the intermediate state must contain a hard on-shall quark and a hard on-shell gluon. But such a state can not be produced by a soft almost on-shell quark in the initial state. Thus this term is also vanishingly small.

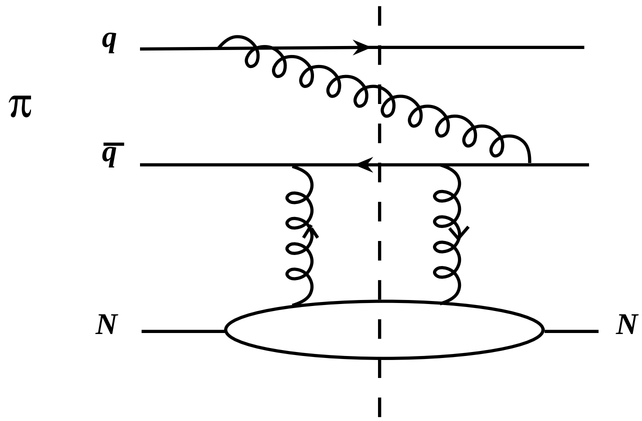

FIG. 3. A time ordering that contributes to T_2. The $q\bar{q}g$ state interacts with the target. Only a single diagram of the several that contribute is shown.

The reasoning of this subsection heavily relied on the ordering in transverse momenta characteristic for the leading order terms in $\alpha_s \ln \kappa_t^2/k_0^2$, where $\kappa_t^2 \gg k_{ti}^2$. Using Ward identities in the way similar to the discussion in the end of subsection B it is easy to demonstrate that in the kinematical region $k_{1t}^2 \approx \kappa_t^2$ diagrams Fig 3-4 do not produce $\ln k_t^2/k_0^2$ and/or $\ln 1/x$ terms. Within this LO approximation it is legitimate to use LO pion wave function-i.e.to account k_t evolution of pion wave function. However calculation of non-leading order (NLO) terms would require a more accurate treatment of NLO corrections to pion wave function. The contribution of the kinematical region $k_{1t}^2 \approx \kappa_t^2$ but small quark transverse momenta in the pion wave function is additionally suppressed by a double logarithmic Sudakov type form factor which accounts for the fact that a violent change of direction of the color current is required for such a term to exist. In the NLO approximation k_{ti} may be as large as κ_t, so $\tilde{m}^2$ would not necessarily be small. Thus our proof that $T_2 = 0$ may be inapplicable beyond the LO approximation.

B. Meson Color Flow Term–T_3

The T_3 or meson-color-flow terms, of Fig. 4 arise from the attachment of both target gluons to the exchanged gluon appearing in V^g as well as the sum of diagrams, Fig. 5, in which one target gluon is attached to potential V^g in the pion wave function and another gluon is attached to a quark in the pion or in the final state. Our reasoning in this sub-section uses Feynman diagrams, so that all time-orderings are included.

The complete sum of all of the terms appearing in the gauge invariant set is suppressed by color coherent destructive interference caused by the color neutrality of the $q\bar{q}g$ intermediate state. We show that the terms of T_3 are reduced by a large factor proportional to $1/\kappa_t^2$ compared with those of T_1. The technical problem in proving this result arises because gluonic interactions include momentum operators, which could bring in large factors arising from longitudinal momenta of the beam.

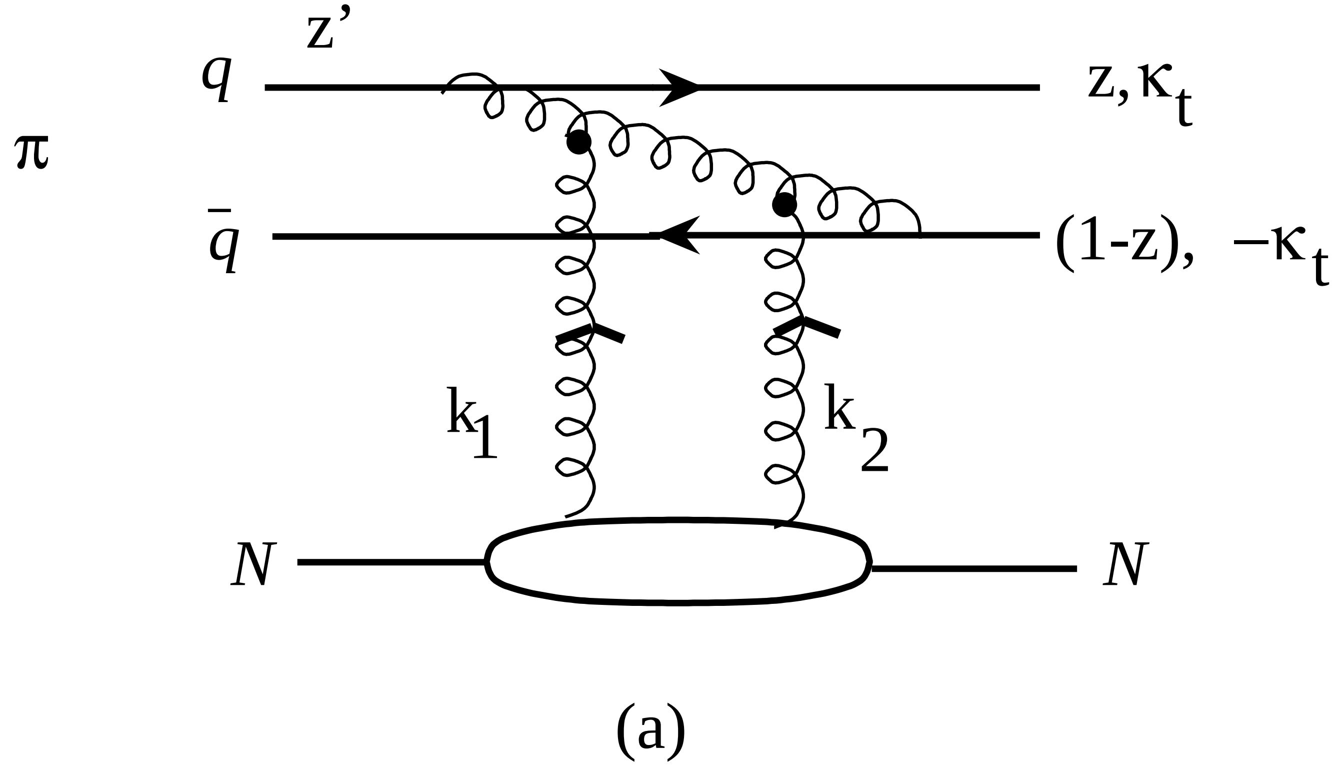

FIG. 4. Contribution to T_3. The exchanged gluon interacts with with each of the two gluons produced by the target. There is also a diagram in which the gluons from the target are crossed, and another two in which the exchanged gluon is emitted by the anti-quark. Only a single diagram of the four that contribute is shown.

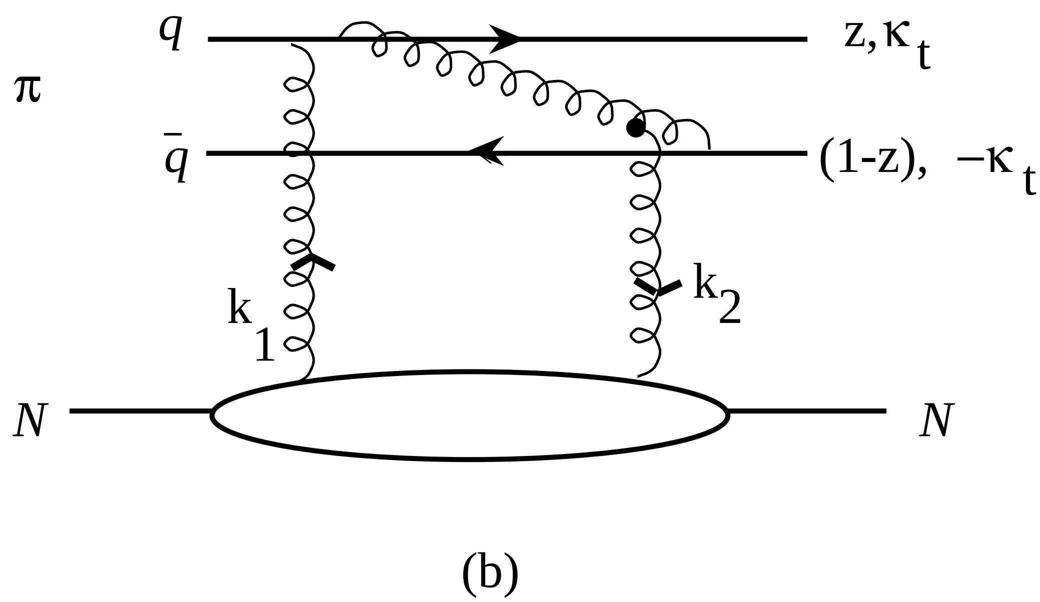

FIG. 5. Contribution to T_3. The exchanged gluon interacts with a gluon of momentum k_2. There is another diagram in which the exchanged gluon is emitted by the anti-quark, and crossed versions of each. The exchanged gluon can also interact with the meson of momentum k_1. Only a single diagram of the eight that contribute is shown.

We show here that the use of QED-type Ward identities [16] (allowed in computing the imaginary part of the diagram, as explained in the Introduction) leads to the result that only the transverse components of the gluon momenta k_1, k_2 enter in the final result for the amplitude T_3. Such momentum factors, which are not large, are included by definition in the skewed gluon distribution.

We examine that part of T_3 arising from the exchange of the gluon k_1, which gives

$$T_3 \propto A^{\pi}_{\mu\nu} d^{\mu\nu} d^{\tilde{\mu}\tilde{\nu}} A^{N}_{\tilde{\mu}\tilde{\nu}} \cdots, \tag{38}$$

where $A^{\pi,N}$ represents the gluon emission amplitude of the pion, nucleon and $d^{\mu\nu}$ arises from the propagator of the gluons emitted or absorbed by the target. At high energies, the gluon propagator can be represented as [17]

$$d^{\mu\nu} \propto \frac{2p^{\mu} p^{\nu}_{\pi}}{2p_{\pi} \cdot p}, \tag{39}$$

so that

$$T_3 \propto \frac{2A^{\pi}_{\mu\lambda} p_{\mu} p_{\lambda}}{2p_{\pi} p} \frac{2A^{N}_{\bar{\mu}\bar{\lambda}} p_{\bar{\mu}} p_{\bar{\lambda}}}{2p_{\pi} p} \tag{40}$$

Now we use current conservation,

$$A^{\pi}_{\mu\lambda} \cdot k_{1\ \mu} = 0, \quad A^{N}_{\bar{\mu}\bar{\lambda}} \cdot k_{1\ \bar{\mu}} = 0, \tag{41}$$

and employ Sudakov variables to describe the momentum k_1:

$$k^{\mu}_1 = \alpha\, p^{\mu} + \beta\, p^{\nu}_{\pi} + k_t. \tag{42}$$

We can determine the quantities α, β by taking the dot product of the above equation with either p or p_{π} and neglecting the relatively small factors of the square of the pion or nucleon mass. This gives

$$\alpha = \frac{k_1 \cdot p_{\pi}}{p \cdot p_{\pi}}, \qquad \beta = \frac{k_1 \cdot p}{p \cdot p_{\pi}}. \tag{43}$$

Using these results in the current conservation relation (41) leads to the relation:

$$A^{\pi}_{\mu\lambda} \cdot p_{\mu} = -\frac{\beta}{\alpha} A^{\pi}_{\mu\lambda} \cdot p_{\pi\ \mu} - \frac{A^{\pi}_{\mu\lambda} \cdot k_{1t\mu}}{\alpha}. \tag{44}$$

The first and third terms of Eq. (44), in difference from the second, are proportional to s and therefore dominate over the second [17]. Thus we find

$$A^{\pi}_{\mu\lambda} \cdot p_{\mu} \approx -\frac{A^{\pi}_{\mu\lambda} \cdot k_{1\ \mu t}}{\alpha}, \tag{45}$$

so that the exchange of the gluon k_1 gives an amplitude (40) proportional to the small transverse momentum k_{1t}. A similar analysis can be performed for the exchange of the gluon k_2, with a result similar to that of Eq. (45). The net result of these considerations is that the contribution of any given diagram, say Fig. 3a, takes the form:

$$T_3 a \propto \frac{4A^{\pi}_{\mu\lambda} \cdot k_{1\ \mu t} k_{1\ \lambda t}}{(2k_1 \cdot p_{\pi})^2} \cdots \frac{4A^{\pi}_{\bar{\mu}\bar{\lambda}} \cdot k_{2\bar{\mu}t} k_{2\bar{\lambda}t}}{(2k_2 \cdot p_{\pi})^2}. \tag{46}$$

The factors involving $k_{i,t}$ will be absorbed into the definition of the skewed gluon distribution of the nucleon. The interesting result is in the denominator of Eq. (46), because

$$2k_i \cdot p = x_i\ s. \tag{47}$$

Conservation of four momentum and taking the intermediate (between the heavy dots) gluon to be on shell gives

$$x_1 = \frac{\kappa_t 2}{(z - z')s}, \tag{48}$$

while the use of Eq. (4) gives

$$x_2 = \frac{\kappa_t^2}{(z - z')s} - \frac{\kappa_t^2}{z(1-z)s}. \tag{49}$$

Both $x_1 s$ and $x_2 s$ are proportional to κ_t^2 unless x_2 vanishes as a result of a cancellation or near cancellation between the two terms of Eq. (49). The condition for this to occur is $z' \approx z^2$. For the values of z far from 0 which are relevant here, the contribution of this region is small because of the smallness of the region of integration over z'. For $z \approx 0$, the end point contribution of points with $z' = 0$ is suppressed by the decrease $\propto z'$ of the pion wave function for small values of z'. The net result is that the sum of terms of T_3 gives $T_3 \propto 1/\kappa_t^6$. Thus T_3 is ignorable.

C. Correction $-T_4$

The T_4 term arises from the sum of Feynman diagrams in which the gluon exchange between the q and $\bar{q}$ in the beam occurs during the interaction with the target, see Fig. 6. The naive expectation is that such terms, which amount to having a gluon exchanged during the very short interaction time characteristic of the two gluon exchange process occurring at high energies, must be very small indeed.

The intent of this sub-section is to use the analytic properties of the scattering amplitude to show that T_4 is negligible. Instead of calculating the sum of the imaginary parts of all of the amplitudes, we will prove that this sum is 0 by analyzing analytic properties of the important diagrams. Each considered diagram contains a product of quark propagators from quarks and (anti-quarks) appearing in the intermediate states. At high energies, the propagators are controlled by the terms of highest power of $x_1\ 2p\cdot p_\pi = x_1\nu$. These propagators have poles in the complex x_1-plane which are located at one side of the contour of integration. The sign of the term containing (ν) in each propagator unambiguously follows from the directions of pion and target momenta. If we can show that the typical integral is of the form

$$\int dx_1 \frac{1}{(\alpha\, x_1\nu - a + i\epsilon)(\beta\, x_1\nu - b + i\epsilon)}, \qquad \alpha, \beta > 0 \tag{50}$$

the proof would be complete, because the techniques of contour integration in which one closes the contour in the upper half plane, lead immediately to the vanishing of the integral.

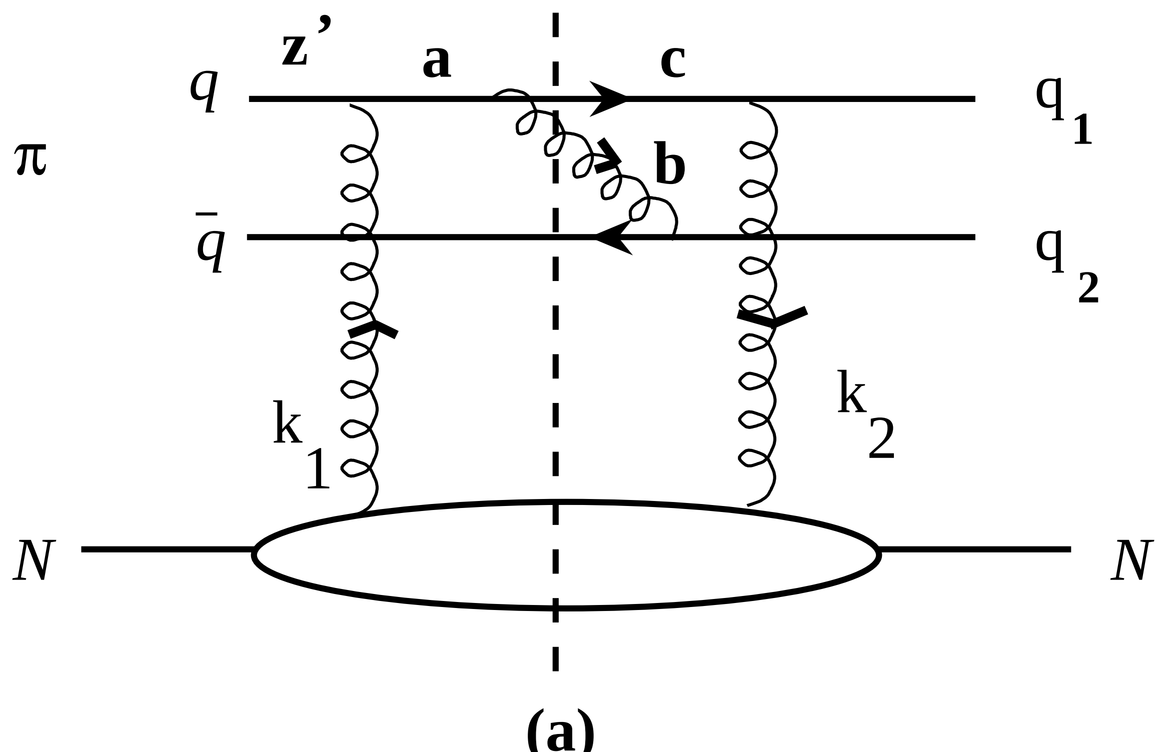

FIG. 6. A contribution to T_4. The quark absorbs a gluon of momentum k_1, exchanges a gluon with the anti-quark, and then emits a gluon of momentum k_2. Only one diagram of the eight that occur is shown.

We now consider the Feynman graphs, starting with Fig. 6. Once again we compute the imaginary part of the graph and consider the intermediate state as being on the energy shell. The propagator for the line (a) has the factor

$$(k_1 + z' p_\pi)^2 - m_q^2 = z' x_1 \nu + \cdots, \tag{51}$$

while that of the line (b) has the form

$$(q_2 - l_2)^2, \tag{52}$$

which is independent of x_1, because q_2 and momentum of quark within pion wave function l are unconnected with target momentum. The propagator of line (c) has the factor

$$(k_2 + q_1)^2 - m_q^2 = x_2\, z\, \nu + \cdots = x_1\, z\, \nu + \cdots. \tag{53}$$

The last equation is obtained from using Eqs. (4,6). The results (51-53) show that the diagram of Fig. 6 takes on the mathematical form of the integral (50). Thus this term vanishes.

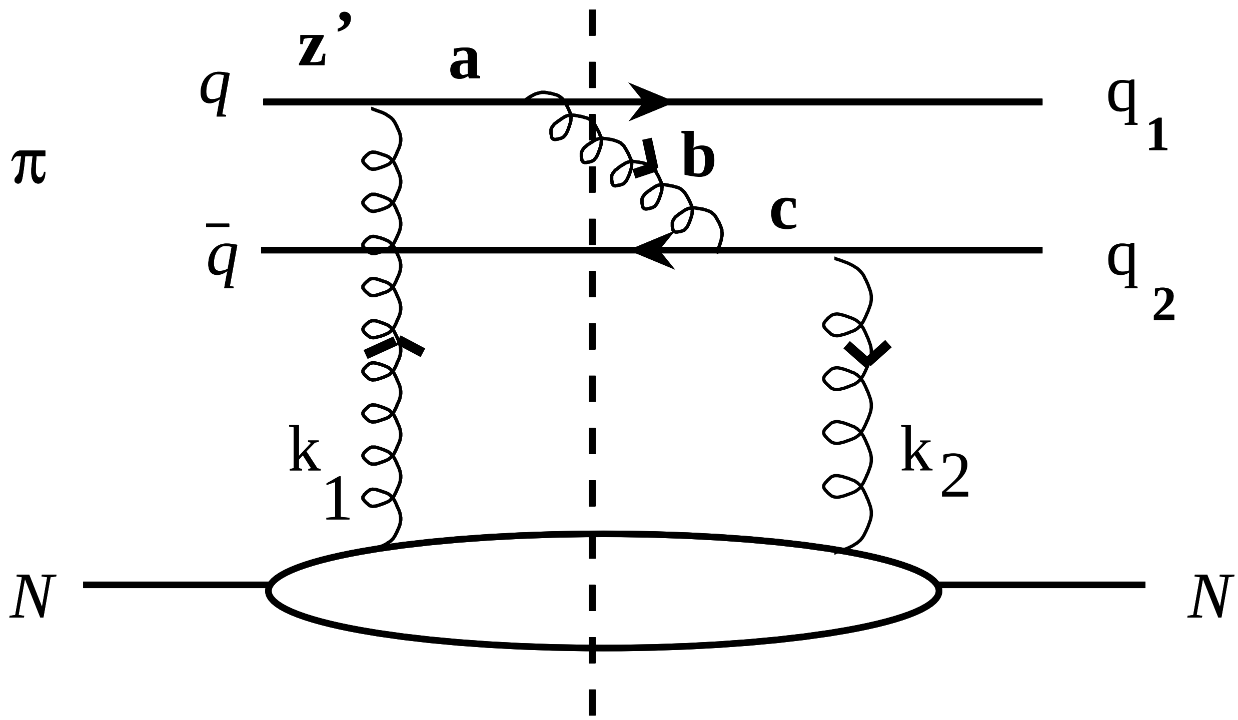

FIG. 7. Another contribution to T_4. The quark absorbs a gluon of momentum k_1, exchanges a gluon with the anti-quark, and the anti-quark emits a gluon of momentum k_2. Only one diagram of the four that contribute is shown.

We also consider the diagram of Fig. 7. In this case there are three propagators (a), (b) (c) which have a term proportional to ν, but the coefficients are not all positive. The propagator factor for line (a) is given by

$$(x_1\, p_\pi + l_1)^2 = x_1 z' \nu + \cdots, \tag{54}$$

while that of line (c) is given by

$$(k_2 + q_2)^2 - m_q^2 = x_2 (1 - z)\nu + \cdots = (1 - z) x_1 \nu + \cdots \tag{55}$$

At the same time coefficient near x_1 in the propagator (b) (gluon production) has no definite sign. Thus for this diagram the integral over x_1 does not vanish. Presence of additional gluon in the intermediate state means that $x_1 \propto x_2$ and repeating the same trick involving gauge invariance we conclude that this diagram is suppressed by the power of κ_t^2 as compared to T_1. Similar logic can be applied to any of the diagrams contributing to T_4. The physical idea that the intermediate $q\bar{q}$ state does not live long enough to exchange a gluon is realized in the ability to close the contour of integration in the upper half plane or in the suppression by the power of k_t^2. The analyticity of the scattering amplitude in the upper half is a consequence of causality. Thus the physical and mathematical ideas behind the vanishing of T_4 are basically equivalent for all diagrams at high enough beam energies.

Feynman diagrams contain also a contribution to diffractive high k_t di-jet production by pion and photon projectiles in which the exchanged gluons have large $k_{1,2t} \approx \kappa_t$ but the quarks in the initial pion have small transverse momenta, and there is no hard interaction between q and $\bar{q}$ in the initial or final states, see Fig. 8. Let us outline various phenomena relevant for the smallness of this contribution.

The term of Fig. 8 is suppressed due to the peculiar kinematics of the gluon exchanges relevant here. To see this, we first apply Eq. (37) to the situation when the transverse momenta of quarks in the initial state, l_t are much smaller than κ_t, so that $k_{1t} \sim \kappa_t, k_{2t} \sim -\kappa_t$. In this case

$$x_2 = \frac{1}{\nu}\left[\frac{\kappa_t^2}{1-\beta-z} - \frac{\kappa_t^2}{z(1-z)}\right]. \qquad (56)$$

In the leading $\log x$ approximation (Multi-Regge kinematics): $\beta \ll 1$ and the condition $x_2 > 0$ cannot be satisfied. Hence this contribution cannot be expressed in terms of the unintegrated gluon density as assumed in [31]. Moreover, to satisfy the condition $x_2 > 0$ one needs

$$1 - z > \beta > (1-z)^2, \text{ or } z > \beta > z^2. \qquad (57)$$

In general the QCD amplitude is given by an integral over β. Thus it follows from Eq. (56) that $x_2\nu \propto k_t^2$ except for a narrow interval in β near $\beta = (1-z)^2$. The contribution of this narrow interval is suppressed by the small length of this interval. (The endpoint $z \sim 0, z \sim 1$ contributions are suppressed by the pion wave function). Application of the same trick, as above, using the Ward identities shows that the contribution of longitudinally polarized gluons which usually dominates is $\propto \kappa_t^{-6}$. Hence we conclude that the contribution of Fig. 8, which was suggested in [31] as a dominant contribution, decreases with κ_t significantly faster than presented in [31] and is strongly suppressed as compared to the leading diagram of Fig. 1.

The intermediate state of this particular contribution corresponds to the kinematics of two color jets with a large rapidity gap. But this is filled in by the radiative effects of perturbative QCD. In PQCD, the probability of a rapidity gap in DIS at not too small values of $k_t^2/x_1\nu$ is negligible because gluon radiation tends to fill the gap. Such effects are precluded in T_1 by the localization of color in transverse space. Thus production of di-jets without gluon radiation should be highly suppressed by Sudakov type form factors. (The natural importance of this diagram occurs for final states with an additional gluon corresponding to jets initiated from components of the pion wave function in which in a pion configuration of average size, the gluons carry $\sim 1/2$ of the pion momentum). Such a process should be strongly suppressed as compared to the amplitude T_1. On the other hand, this process is allowed only if if quark transverse momenta within the $q\bar{q}$ pair are $\approx \kappa_t$. This means that such a contribution could occur in $\gamma^* N$ scattering at small impact parameter $b^2 \propto 1/k_t^2$. Such a scattering is feasible, if the transverse size of q$\bar{q}$ pair is also $\propto b^2$.

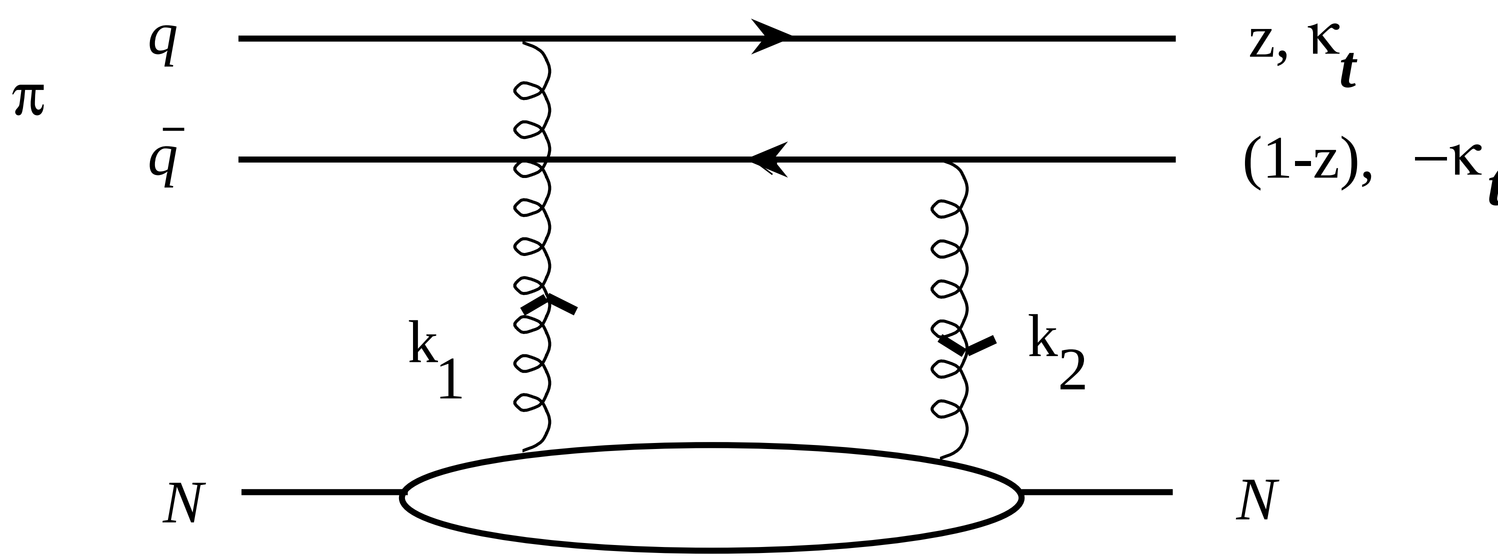

FIG. 8. A negligible contribution.

IV. NUCLEAR DEPENDENCE OF THE AMPLITUDE

The picture we have obtained is that the amplitude is dominated by a process in which the pion becomes a $q\bar{q}$ pair of essentially zero transverse extent well before hitting the nuclear target. This point like configuration PLC can move through the entire nucleus without expanding. The $q\bar{q}$ can interact with one nucleon and can pass undisturbed through any other nucleon. For zero momentum transfer, q_t, to the nucleus, the amplitude $\mathcal{M}$ (A) takes the form

$$\mathcal{M}(\mathrm{A}) = \mathrm{A}\ \mathcal{M}(N)\frac{G_A(x_1,x_2,m_f^2)}{AG_N(x_1,x_2,m_f^2)}\left(1+\frac{\epsilon}{< b^2 > \kappa_t^2}\mathrm{A}^{1/3}\right) \equiv \mathrm{A}\mathcal{M}(N)\Gamma\ , \tag{58}$$

in which the skewed-ness of the gluonic distribution is made explicit, and where the real number $\epsilon > 0$ and $\Gamma > 1$. Observe the factor A which is the dominant effect here. This factor is contained in the ratio of gluon distributions in a nucleus and in a nucleon [3]. This dependence on atomic number is a reliable prediction of QCD in the limit $m_f^2, s \to \infty$ with fixed x_N (of Eq. (28)). On the contrary, for $x_N \to 0$ with fixed m_f^2, the nuclear shadowing of gluon distribution in a nucleus becomes very important [3,29].

The ϵ correction term in Eq. (58) arises from a single soft re-scattering which can occur as the PLC moves through the nuclear length ($R_A \propto \mathrm{A}^{1/3}$). That $\epsilon > 0,\ \Gamma > 1$ was a somewhat surprising feature of our 1993 calculation because the usual second order scattering, as treated in Glauber theory, always reduces cross sections. The key features of the usual first order term are: $f = i\sigma_0$, and those of the usual second order term are $if^2 = -i\sigma_0^2$. Note the opposite signs. For us here $f = i\sigma_0 b^2/ < b^2 >$, which for very large values of k_t^2 becomes $f = -i\sigma_0/(< b^2 > \kappa_t^2)$. The second order term depends on $if^2 = i[-i\sigma_0^2/(< b^2 > \kappa_t^2)^2]^2$, which now has the same sign as the first–order term.

The differential cross section takes the form

$$\frac{d\sigma(A)}{dq_t^2} = A^2\Gamma^2\frac{d\sigma}{dt}(N)e^{tR_A^2/3}\ , \tag{59}$$

for small values of t. Note that

$$-t = q_t^2 - t_{\min}, \tag{60}$$

where $-t_{\min}$ is the minimum value of the square of the longitudinal momentum transfer:

$$-t_{\min} = \left(\frac{m_f^2 - m_\pi^2}{2p_\pi}\right)^2 \tag{61}$$

Our discussion below is applicable in the kinematics where $-t_{min}R_A^2/3 \leq 1$ so that the entire dependence of the cross section on t_{min} is contained in the factor $e^{t_{min}R_A^2/3}$. One measures the integral

$$\sigma(A) = \int dt\frac{d\sigma(A)}{dt} = \frac{3}{R_A^2}A^2\Gamma^2\sigma(N)\ . \tag{62}$$

A typical procedure is to parameterize $\sigma(A)$ as

$$\sigma(A) = \sigma_1 A^\alpha \tag{63}$$

in which σ_1 is a constant independent of A. For the R_A corresponding to the two targets Pt (A = 195) and C(A = 12) of E791, one finds $\alpha \approx 1.45$. The experiment [6] does not directly measure the coherent nuclear scattering cross section. This must be extracted from a measurement which includes the effects of nuclear excitation. The extraction is discussed below.

As pointed out previously [8], the values of our multiple scattering correction- Γ of our 1993 calculation [3] were overestimated by an factor of approximately four. This is because the σ_0 was chosen to be larger by a factor of 4 than in [21] and in Eq. (27). We now find that for values of κ_t greater than about 2 GeV, the coefficient α could be enhanced by between 0.0 and 0.08, depending on the value of κ_t. Taking 0.04 as a mean value one finds $\alpha \approx 1.5$. This estimate depends on the use of a model for the non-perturbative part of $|\pi_{q\bar{q}}\rangle$, and also on the validity of a simple eikonal treatment for the multiple-scattering corrections which is questionable at the high energies that we consider here.

V. EXPERIMENTAL CONSIDERATIONS

The requirements for observing the influence of color transparency were discussed in 1993 [3]. The two jets should have total transverse momentum to be very small. The relative transverse momentum should be greater than about ≥ 2 GeV/c and the mass of diffractive state should be described by the formulae:

$$m_f^2 = \frac{m_q^2 + \kappa_t^2}{z(1-z)} \tag{64}$$

The last requirement is important to suppress diffraction into a $q\bar{q}g$ pair in which the gluon transverse momentum is not too small.

It would be nice if one could measure the jet momenta precisely enough so as to be able to identify the final nucleus as the target ground state. But this is impossible for the energies we are concerned here. Another technique must be used. The technique used in [6] is to isolate the dependence of the elastic diffractive peak on the momentum transfer to the target, t, as the distinctive property of the coherent processes. The amplitude for the excitations of low-lying even-parity nuclear levels $\sim -t$, due to the orthogonality of the ground and excited state nuclear wave functions. Thus the cross section of these kinds of soft nuclear excitations integrated over t is suppressed by an additional factor of $1/R_A^4 \approx A^{-4/3}$ compared to the nuclear coherent process. For $\sqrt{-t}\, R_A \gg 1$ where q is the momentum transfer to the nucleus $q = \sqrt{-t}$. The background processes involving nuclear excitations vary as A, so an unwanted counting of such would actually weaken the signal we seek. For $qR_A \gg 1$ diffractive peak can not be used as signature of diffractive processes to distinguish them from non-diffractive processes whose cross section $\propto \sigma(\pi N) \propto A^{0.75}$. Thus substantial A-dependence, $\sigma \propto A^\alpha$, with $\alpha \approx 1.5$, as predicted by QCD for large enough values of κ_t should be distinguishable from the background processes. To distinguish between diffraction into $q\bar{q}$ and $q\bar{q}g$ states, it is useful to explore the formulae for the mass of two jet system expressed in terms of transverse momenta of jets and fraction of pion momentum carried by jet. The $q\bar{q}g$ states have larger mass and a different distribution over k_t and z than the $q\bar{q}$ states.

The amplitude varies as $\mathcal{M}(A) \sim \alpha_s/\kappa_t^4$

$$\mathcal{M}(A) \sim \alpha_s x_N G_N(x_N, Q_{eff}^2)/\kappa_t^4, \tag{65}$$

where $Q_{eff}^2 \sim 2\kappa_t^2$. For the kinematics of the E791 experiment, where x_N increases $\propto \kappa_t^2$, the factor $\alpha_s x_N G_N(x_N, Q_{eff}^2)$ is a rather weak function of κ_t. For example, if we use the standard CTEQ5M parameterization we find $\sigma(A) \sim 1/\kappa_t^{8.5}$ for $1.5 \leq \kappa_t \leq 2.5$ GeV/c which is consistent with the data [6].

For the amplitude discussed here, $\sigma(A) \sim (z(1-z))^2$ which is in the excellent agreement with the data [6].

A. Extracting the coherent contribution

The experiment is discussed in Ref. [6]. The main advantage of this experiment is the excellent resolution of the transverse momentum. The reference also shows the identification of the di-jet using the Jade algorithm, and displays the identification of the diffractive peak by the q_t^2 dependence for very low q_t^2. This dependence is consistent with that obtained from the previously measured radii $R(C) = 2.44$ fm, and $R_{Pt} = 5.27$ fm. The key feature is the identification of the coherent contribution from its rapid fall-off with t.

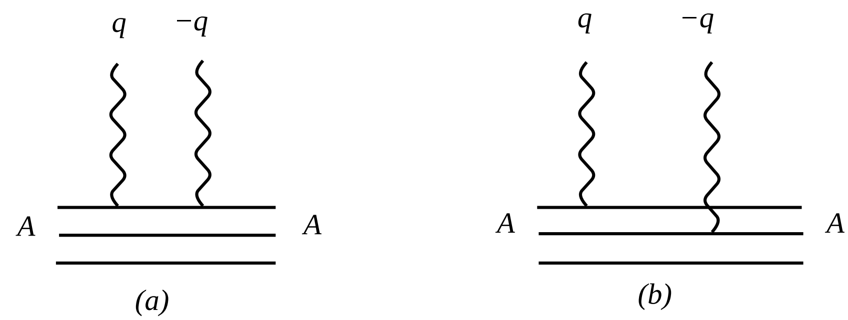

FIG. 9. Contributions to the total nuclear diffractive cross section. (a) The terms with $i = j$. (b) $i \neq j$.

We discuss the extraction of this signal in some detail. We consider the contribution to the total cross section $\frac{d\sigma_A}{dt}$ that arises from diffractive production of the di-jet. The total cross section includes terms in which the final nucleus is not the ground state. The excitation energy is small compared to the energy of the beam, so that one may use closure to treat the sum over nuclear excited states. Then the cross section is evaluated as a ground state matrix element of an operator $\sum_{i,j} e^{i\mathrm{q}\cdot(\mathrm{r}_i - \mathrm{r}_j)} = A + \sum_{i\neq j} e^{i\mathrm{q}\cdot(\mathrm{r}_i - \mathrm{r}_j)}$; see Fig. 9. The result, obtained by correctly using r_i relative to the nuclear center of mass and by neglecting correlations in the nuclear wave function is given by:

$$\frac{d\sigma_A}{dt} = \left[A + A(A-1)F_A^2\left(t\,\frac{A}{A-1}\right)\right]\frac{d\sigma_N}{dt}. \tag{66}$$

The factor $\frac{A}{A-1}$ in the argument of F_A is due to accountinf for nuclear recoil in the mean field approximation, cf [32]. This formula should be very accurate, for the small values of t relevant here. The coherent term of our interest is

$$\frac{d\sigma_N}{dt} = A^2\ F_A^2(t)\frac{d\sigma_N}{dt}, \tag{67}$$

and the contribution of excited nuclear states is the difference: $\frac{d\sigma_A}{dt} - \frac{d\sigma_N}{dt}$, which vanishes at $t = 0$. The experiment proceeds by removing a term $\propto A$ from $\frac{d\sigma_A}{dt}$ which has no rapid variation with t. This defines a new cross section

$$\frac{d\tilde{\sigma}_A}{dt} = A(A-1)F_A^2\left(t\,\frac{A}{A-1}\right)\frac{d\sigma_N}{dt}. \tag{68}$$

The integral of this term over t can be extracted from the data:

$$\sigma_1 \equiv \int dt\ \frac{d\tilde{\sigma}_A}{dt} = \frac{(A-1)^2}{R_A^2}. \tag{69}$$

Note that this differs by a factor of $\frac{(A-1)^2}{A^2}$ from the A-dependence predicted by color transparency for coherent processes, recall Eq. (59). The parameterization $\sigma \propto A^\alpha$ gives then

$$\alpha = 1.51 \tag{70}$$

instead of $\alpha = 1.45$, if one uses $A = 12, 195$.

The result [6] of the experiment is

$$\alpha \approx 1.55 \pm 0.05 \ , \tag{71}$$

which is remarkably close to the theoretical value shown in Eq. (70). The size of our multiple scattering correction, discussed in the previous Section, is of the order of the experimental error bar.

VI. ELECTROMAGNETIC BACKGROUND

Because very low values of q_t^2 are involved, one could ask if the process occurs by one photon exchange (a type of Primakoff effect) instead of two gluon exchange. If the momentum transfer is very low, the process is peripheral and there would be no initial or final state interactions. Thus, it is necessary to estimate the relative importance of the two effects.

This amplitude is caused by the exchange of a virtual photon of four-momentum q ($q^2 = t$) with the target. The nuclear Primakoff amplitude is then given by

$$\mathcal{M}_P(A) = e^2 \frac{\langle \pi | J_\mu^{\rm em} | q\bar{q} \rangle}{-t} (P_A^i + P_A^f)^\mu \frac{Z}{A} F_A(t) \approx 2e^2 \langle \pi | J^{\rm em} \cdot \frac{P_A}{A} | q\bar{q} \rangle \frac{Z F_A(t)}{-t}, \tag{72}$$

where we may use the decomposition $-t = q_t^2 - t_{\rm min}$. A photon can be attached to any charged particle, so a direct calculation would involve a complicated sum of diagrams. We may simplify the calculation by using the feature that the electromagnetic current is conserved. This application is simplified by the use of Sudakov variables, which is a necessary first step. Accordingly, we write

$$q = \alpha \frac{P_A}{A} + \beta p_\pi + q_t. \tag{73}$$

Conservation of four momentum gives

$$\beta = \frac{q^2}{2(p_\pi \cdot P_A)}, \quad \alpha = \frac{-m_f^2}{s}. \tag{74}$$

Then conservation of current can be written as

$$\langle \pi | J^{\rm em} \cdot q | q\bar{q} \rangle \approx \alpha \langle \pi | J^{\rm em} \cdot \frac{P_A}{A} | q\bar{q} \rangle + \beta \langle \pi | J^{\rm em} \cdot p_\pi | q\bar{q} \rangle - \langle \pi | J^{\rm em} \cdot q_t | q\bar{q} \rangle = 0. \tag{75}$$

The use of Eq. (74) allows us to neglect the β term of Eq. (75) so that

$$\alpha \langle \pi | J^{\rm em} \cdot \frac{P_A}{A} | q\bar{q} \rangle = \langle \pi | J^{\rm em} \cdot q_t | q\bar{q} \rangle \tag{76}$$

By definition, the transverse momentum of pion is zero so the dominant contribution in Eq. (76) is given by photon attachments to quark lines, and the matrix element is given by

$$\langle \pi | J^{\rm em} \cdot q_t | q\bar{q} \rangle = \psi_\pi(x, \kappa_t) q_t \cdot \kappa_t. \tag{77}$$

The net result, obtained by using Eq. (76) in Eq. (72), is

$$\mathcal{M}_P(A) = \frac{-e^2 \psi_\pi(z, \kappa_t) Z}{q_t^2 - t_{\rm min}} F_A(t) \frac{s}{m_f^2} 2 q_t \cdot \kappa_t \ , \tag{78}$$

which should be compared with the amplitude of Eq. (31) (including the effect of the nuclear form factor, which enters at non-zero values of t, but ignoring the logarithmic correction) written as

$$\mathcal{M}(A) = -\psi_\pi(z, \kappa_t) A \frac{s\sigma_0}{< b^2 >} F_A(t) 4 \tag{79}$$

The ratio of electromagnetic and strong amplitudes is given by

$$\frac{\mathcal{M}_P(A)}{\mathcal{M}(A)} = -i\, e^2\, \frac{Z}{A}\, \frac{z(1-z)}{\sigma_0/<b^2>}\, \frac{q\cdot\kappa_t}{2(q_t^2 - t_{\min})} \tag{80}$$

Using $q_t^2 \approx 0.02$ GeV^2 (the smallest value measured in Exp. 791), Z/A=1/2, $e^2 = 4\pi/137, \kappa_t = 2$ GeV, $\sigma_0/<b^2>\approx 2.5$ (this is 1/4 the value of Ref. [3]), $z = 1/2$, and taking q_t parallel to κ_t we find that

$$\frac{\mathcal{M}_P(N)}{\mathcal{M}(N)} \approx -0.03\, i. \tag{81}$$

Thus the Primakoff term is very small and, because of its real nature, does not interfere with the larger strong amplitude. We may safely ignore this effect at fixed target energy range.

For heavy nuclei target another electromagnetic process $\propto Z^2\alpha_{em}^2$ in the amplitude gives contribution. This is di-jet production due to two photon exchange, a version of Fig. 8 in which the exchanged gluons are replaced by exchanged photons. For small transverse momenta of quarks $l_t^2 \ll k_t^2$ in the pion wave function, this contribution is suppressed as the power of x in the amplitude. This is because, in the calculation of imaginary part of diagram, x_2 for exchanged photon is given

$$x_2\nu = -\frac{k_t^2}{z} \tag{82}$$

except for very narrow region of z near $z = 0, 1$ which is suppressed by the decrease of pion wave function. Thus $x_2 < 0$ and according to our previous arguments, this contribution should be very small. For large l_t this contribution may also be expressed in terms of the same pion wave function as in the case of two gluon exchange. It is easy to estimate the ratio of this EM amplitude to the strong amplitude:

$$\frac{Z^2\alpha_{em}^2 \int \frac{d^2k_{1t}}{(2\pi)^2} F_A(k_{1t}^2) F_A((k_{1t}-q_t)^2) \frac{k_{1t}^2}{(k_{1t})^2 - t_{min})(k_{1t}-q_t)^2 - t_{min})}}{\frac{4}{3} A\alpha_s F_A(q_t) x_1 G_A(x_1, x_2, \kappa_t^2)} \tag{83}$$

Thus the contribution of this term to the cross section should have the same z and κ_t^2 dependence as the two gluon exchange term, but with much faster Z dependence ($\propto Z^4$) and slower decrease with q_t. This term is negligible also.

VII. DISCUSSION AND SUMMARY

The use of the experimentally measured [6] value of $\alpha = 1.55$ (recall Eq. (63)) leads to

$$\frac{\sigma(Pt)}{\sigma(C)} = 75\;. \tag{84}$$

The typical usual nuclear dependence of soft diffractive processes observed in high energy processes is $\approx A^{2/3}$ or $\alpha = 2/3$. The use of Glauber approximation with typical hadronic cross section for the final system tends to predict A dependence as $\approx A^{1/3}$, account of color fluctuations [33] predicts $\approx A^{2/3}$, in agreement with the FNAL data [34] which would give

$$\frac{\sigma_{\rm USUAL}(Pt)}{\sigma_{\rm USUAL}(C)} = 7. \tag{85}$$

Thus color transparency causes a factor of 10 enhancement! This seems to be the huge effect of color transparency that many of us have been hoping to find. It is also true that, as noted in the Introduction, that the κ_t and z dependence of the cross section [6] is in accord with our prediction.

All of this looks very good, but it is necessary to provide some words of caution. Our analysis related to a nuclear coherent process involving a $q\bar{q}$ final state. If the experimental signal is significantly contaminated by incoherent nuclear effects or by $q\bar{q}g$ final states, our analysis might not be applicable. However, the experimental [6] extraction of the coherent peak using the q_t^2 dependence of the amplitude, and the measurement of the 2 jet (as opposed to three jet cross section) seem very secure to us, except for the small correction discussed in Sect. V. Another worry is that the color transparency effect seen in Ref. [6] seems to start for values of κ_t near 1 GeV. These are lower than suggested in Ref. [3]. These earlier predictions used modeling of non-perturbative effects, and such modeling may be necessary to guess the lowest values of κ_t for which color transparency would occur. The reasoning of the present paper uses perturbative QCD, which becomes more reliable as κ_t increases. This is because the competing amplitudes $T_{2,3,4}$ are decreased relative to T_1 by a factor of $\frac{\Lambda^2}{\kappa_t^2}$. This ratio is $\approx .04$ for $\kappa_t = 1$ GeV, so a coherent sum of the sub-dominant amplitudes could provide a significant correction to our dominant pure amplitude. However, the observed fall-off of the cross section with κ_t, combined with the z and A dependence does provide very strong evidence for color transparency.

If color transparency has been correctly observed in $\pi+A \to q\bar{q}+A$ (ground state), there would be many implications. The spectacular enhancement of the cross section would be a new novel effect. The point like configurations PLC would be proved to exist. This would be one more verification of the concept and implications of the idea of color. Furthermore, the definitive proof of the existence of color transparency means that we now have available a new effective tool to investigate microscopic hadron, nuclear structure at lower energies of a ten's of GeV. In particular, previous experiments [35] showing hints [36] of color transparency (for a review see Ref [4]) probably do show color transparency. Efforts [37] to observe color transparency at Jefferson Laboratory should be increased.

The observation of CT confirms the idea that the life span of the perturbative phase can be increased by the large Lorentz factor associated with high energy beams. A challenging problem would be to explore this idea to observe the perturbative phase in a "macroscopic" volume. One manifestation of this would be the production of huge blob–like configurations of Huskyons [38]. These different configurations have wildly different interactions with a nucleus [39], so that the nucleon in the nucleus can be very different from a free nucleon. More generally, the idea that a nucleon is a composite object is emphasized by these findings. Some configurations of the nucleon interact very strongly with the surrounding nucleons, some interact very weakly. This means that the nucleon in the nucleus can be very different from a free nucleon. This leads to an entirely new view of the nucleus, one in which the nucleus is made out of oscillating, pulsating, vibrating, color singlet, composite objects.

The technical purpose of this paper has been to show how to apply leading-order perturbative QCD to computing the scattering amplitude for the coherent processes: $\pi N \to JJ\ N$ and $\pi A \to JJ\ A$. The high momentum component of the pion wave function, computable in perturbation theory is an essential element of the amplitude. The dominance of the amplitude of the T_1 term of Eq. (31), is obtained by showing that the corrections to it, which at first glance seem to be of the same order in the coupling constant, are vanishingly small. This vanishing, obtained using arguments based on analyticity, causality, and current conservation, is equivalent to the verification of a specific space-time description of the event: the pion produces its point-like component at distances well before the target. Furthermore, for the conditions of the experiment [6] studied here, the competing electromagnetic production process is shown to yield a negligible effect. It seems that perturbative QCD can be applied to the coherent nuclear production of high-relative momentum di-jets by high energy pions.

It therefore seems interesting to consider similar reactions involving other projectiles such as photons, kaons and protons. The observations of the coherent photo-production of the J/Ψ from nuclear targets has long been known [40] to have an A-dependence which is very similar to that observed here, but there seems to have been no theoretical analysis of this process within the framework of QCD. Our present theory can be used for kaon projectiles with little modification. Because the kaon has smaller size than the pion, we expect that the amplitude for a kaon induced process should be somewhat larger than that of the pion induced process discussed here. The study of high energy coherent production of jet systems from nuclear targets seems to be a very productive way to investigate both features of perturbative QCD and microscopic nuclear structure by exploring color transparency phenomenon.

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